

Introduction

This paper discusses some of the ways in which computers can be used in the production of information for the construction of objects from aeroplanes to gridshell structures. This raises some interesting questions concerning the education of future engineers and architects.

Figure 1 shows a 'sinusoidal spiral' and the computer program which produced it. The program is written in International Standards Organisation C++ and will run on any computer with a C++ compiler (Unix, Linux, PC, Apple etc.). It produces a dxf file which is admittedly an Autodesk (AutoCAD) standard, but can be opened by MicroStation, VectorWorks and many other applications.

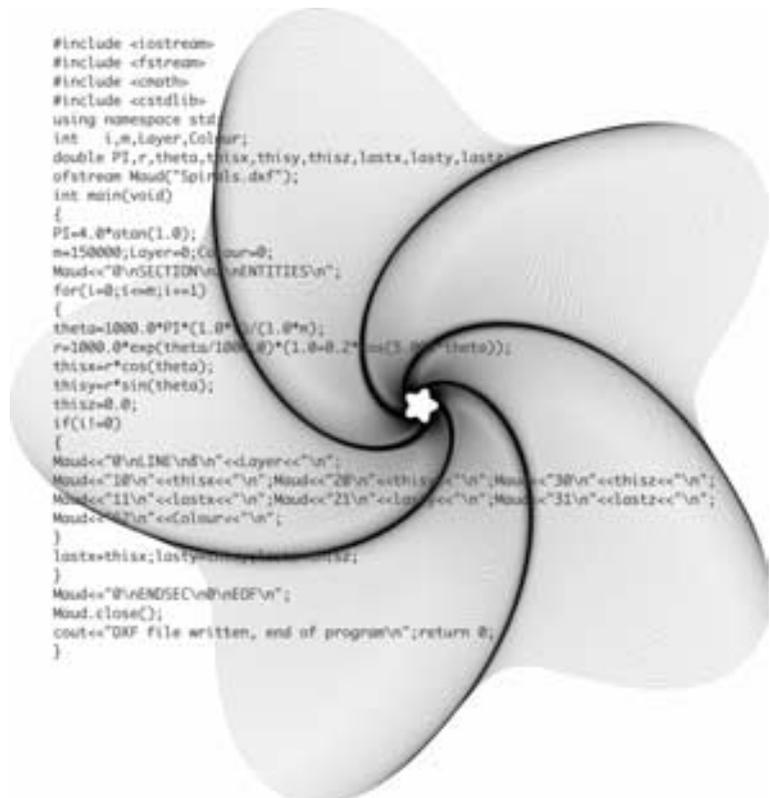


Fig. 1. Sinusoidal spiral and C++ program

This figure raises two issues. Firstly, if such a simple program can produce such a complex result, why is computer programming never taught to architects? Secondly, how important is it to try and promote the use of software which is under the control of bodies like the International Standards Organisation rather than commercial organisations?

Graphics software comes from two main sources: the entertainment industry (films, computer games etc.) and the aerospace and automobile industries. Software from the entertainment industry is ideal for producing images and sketch designs, but not for the production of detailed information for construction. Following the discovery of B-splines by I. Schoenberg in 1946, CAGD (Computer Aided Geometric Design) was first developed in 1960's by many people including Paul de Faget de Casteljau at Citroën, Pierre Bézier at Renault, J. Ferguson at Boeing and C. de Boor at General Motors.

Even though this paper is about the use of computers in design, one should not overstate their importance. The objects in figures 2 to 8 were all designed without computers, except, perhaps for the Boeing 707. The Citroën DS and the Boeing 707 were designed in the 1940's and early 1950's and the Boeing 747 (figure 9) first flew in 1969, a few months after the moon landing. Computers would have been used in the design of the 747, but they would have been much less powerful than the cheapest computer available today.

Computers have no intelligence but enormous calculating power. Humans, and other animals, have enormous intelligence, but limited calculating



Fig. 2. Masonry aqueduct



Fig. 3. Pont du Garabit, Léon Boyer, Maurice Koechlin, Gustave Alexandre Eiffel



Fig. 4. King's College Chapel, Cambridge



Fig. 5. Palm house Kew Gardens, Decimus Burton and Richard Turner

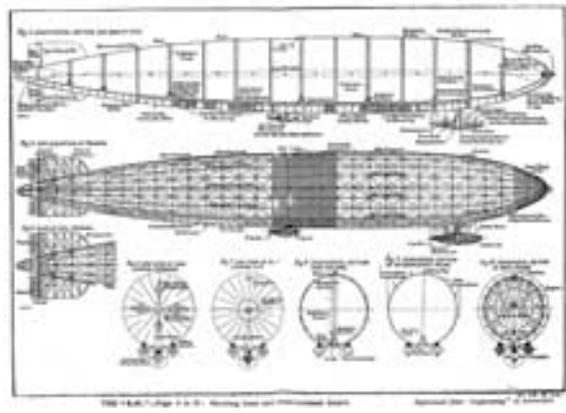


Fig. 6. R80



Fig. 7. Citroën ID19 (similar to the DS)



Fig. 8. Boeing 707



Fig. 8. Boeing 747

power in terms of arithmetic. But just walking about requires the analysis of all sorts of data from the senses and the control of innumerable muscles. This is way beyond the most powerful computers with the most sophisticated software. All that computers can do is to follow simple rules quickly and reliably. A piece of software may contain thousands of rules and this gives an illusion of intelligence.

One of the first uses of computers was for the analysis of structures, using theories that have been developed continuously from the 16th century¹ (figure 10). As a student in 1970, I was taught computing on an IBM 1130 (figure 11) and more time was spent teaching programming to engineering students then than now. This is because the assumption then was that engineers would write their own programs, whereas now the assumption is that they will buy them. Day to day calculations were done on a slide rule, (figure 12) which was the engineer's badge of office, like a doctor's stethoscope. Pocket calculators were introduced in the early 1970's and the slide rule was obsolete by the end of the decade.

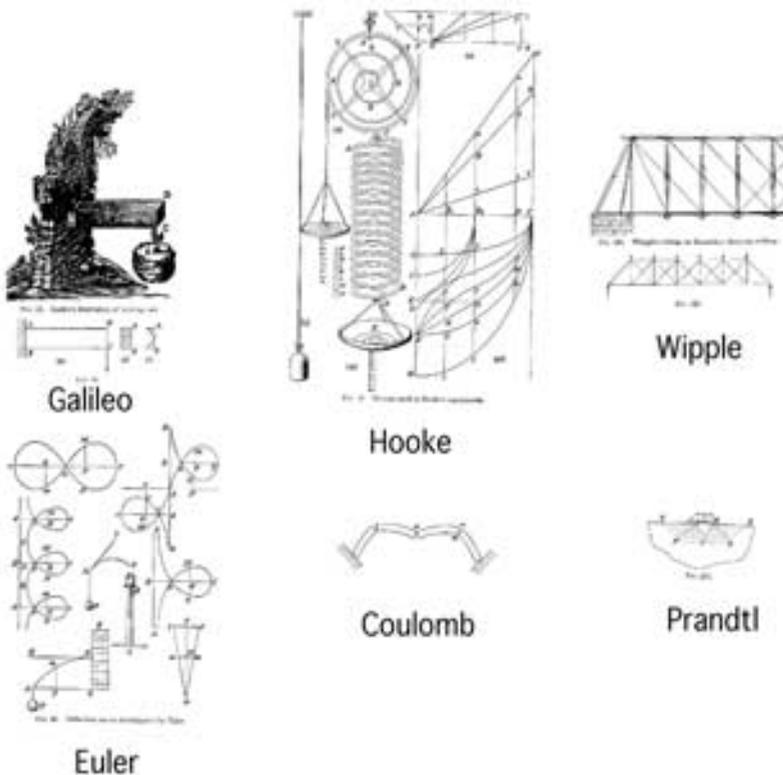


Fig. 10. History of Strength of Materials, Stephen P. Timoshenko



Fig. 11. IBM 1130

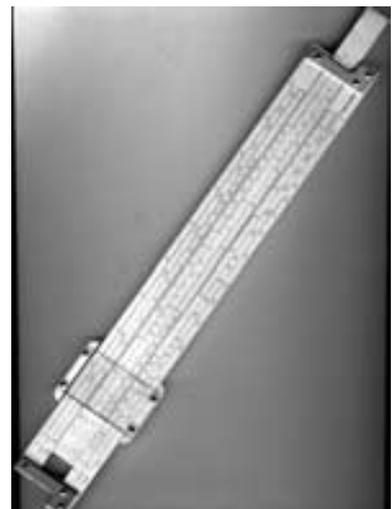


Fig. 12. Slide rule

The Oxford English Dictionary ² defines an algorithm as 'A process, or set of rules, usually one expressed in algebraic notation, now used especially in computing, machine translation and linguistics.' Each rule of an algorithm must be open to only one possible interpretation, which means that no intelligence is required in using the rule.

When a person designs an object they will consciously or unconsciously adopt a set of rules. These may be some rules of proportion or the principles of structures or fluid mechanics or limitation on cost or the materials available. The rules are extremely unlikely to be in the form of an algorithm, they will be vague, incomplete, contradictory, open to dispute and require a great deal of intelligence to interpret. One of the main functions of the professions is to make sure that their rules are so complicated that only their members and their expensive software can interpret them.

Running the same program will always produce the same result, even if it contains a random number generator, unless the program is 'seeded' by some number that is never repeated, like the date and time. However, the first time a program is run the result may not be predictable, because a change to one rule out of thousands may have far reaching effects.

So, given that one can only expect an algorithm to produce a design for one aspect of a complex object, how can one proceed to construct an algorithm? One possibility is to mimic some rule of the nature as pioneered by Gaudí in his hanging tension models that were inverted for his compression vault structures. Frei Otto continued this work in his experiments on hanging chains and soap films (figure 13). These techniques lead to the design of the Mannheim grid shells (figures 14 and 15) and the Munich Aviary (figure 16). Even though physical modelling was used for these projects, in the end their geometry and structural action was determined by computer analysis. At this time, in the 1970's, there was a fierce debate, particularly in Germany, between the more free thinking model makers and the computer programmers. This debate is now over and sketch models are used for initial design, but all final fabrication information is computerised.

Algorithms

Physical Analogies



Fig. 13 Frei Otto, Institut für leichte Flächentragwerke (IL)

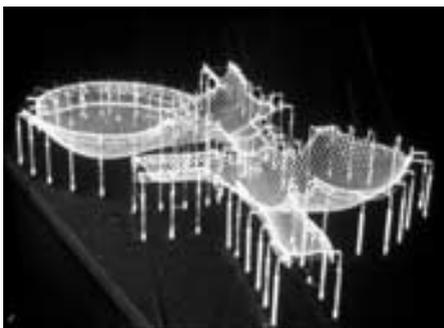


Fig. 14. Mannheim Bundesgartenschau hanging model, Frie Otto, Ove Arup (Happold, Liddell, Williams)



Fig. 15 Mannheim load test



Fig. 16. Munich Zoo Aviary, Frei Otto, Buro Happold, Mike Barnes

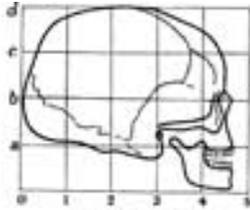


Fig. 177. Human skull.



Fig. 178. Co-ordinates of chimpanzee's skull, as a projection of the Cartesian co-ordinates of Fig. 177.



Fig. 179. Skull of chimpanzee.

Fig. 17. From 'On growth and form' by D'Arcy Thompson

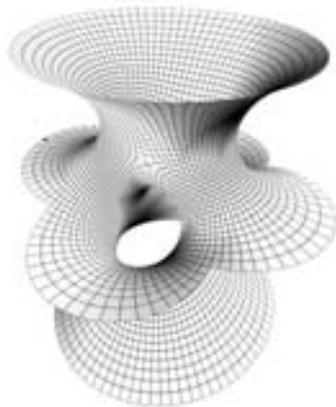


Fig. 18. Costa Minimal Surface

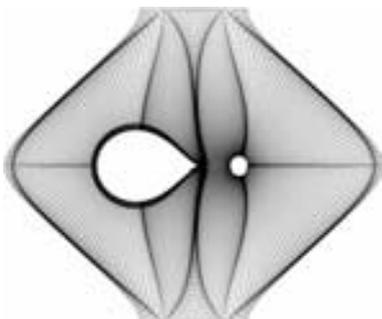


Fig. 20. Shell structure

A similar debate is now taking place between over the use of wind tunnels or of computational fluid dynamics in the analysis of wind loads on structures. In the end the computational approach is bound to win.

The work of D'Arcy Thompson³ (figure 17) is a continuing inspiration for architects and engineers interested in the physical forces driving the form of plants and animals. Figure 17 shows conformal mapping which is a topic intimately connected to minimal surfaces (soap films) through

$$i = \sqrt{-1}$$

A recently discovered minimal surface is the Costa surface⁴⁵ (figure 18). This surface is described by

$$x = \frac{1}{2} \Re \left\{ -\zeta(w) + \pi w + \frac{\pi^2}{4e_1} + \frac{\pi}{2e_1} \left[\zeta \left(w - \frac{1}{2} \right) - \zeta \left(w - \frac{i}{2} \right) \right] \right\},$$

$$y = \frac{1}{2} \Re \left\{ -i\zeta(w) - i\pi w + \frac{\pi^2}{4e_1} - \frac{\pi}{2e_1} \left[i\zeta \left(w - \frac{1}{2} \right) - i\zeta \left(w - \frac{i}{2} \right) \right] \right\}$$

and

$$z = \frac{1}{4} \sqrt{2\pi} \log \left| \frac{\wp(w) - e_1}{\wp(w) + e_1} \right| \text{ where } w = u + iv, \wp \text{ is the Weierstrass}$$

elliptic function⁶ and $\frac{d\zeta(w)}{dw} = -\wp(w)$. The lines on the surface are in the directions of the principal curvatures given by lines of constant α

$$\text{and } \beta \text{ where } \alpha + i\beta = \int \sqrt{\frac{1}{2} + \frac{\wp^2(w)}{\wp^2(w) - e_1^2}} dw.$$

The catenoid and helicoid are both minimal surfaces and a surface can be continuously bent from one to the other without stretching and at the same time remaining a minimal surface (figure 19).



Fig. 19. Bending of catenoid to helicoid

Figures 20 to 25 are design studies produced using complex analytic functions again related to conformal mapping and $\sqrt{-1}$.



Fig. 21. Filigree bridge

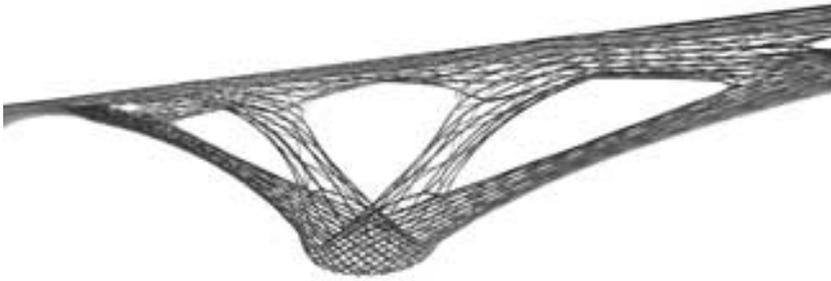


Fig. 22. Tube bridge



Fig. 23. Slender bridge



Fig. 24. Conformal map roof

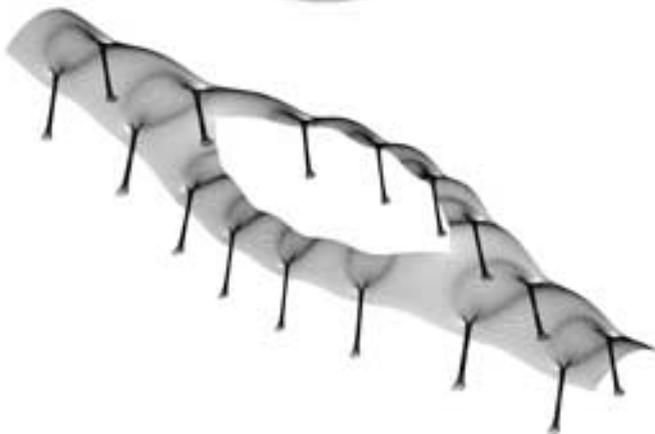
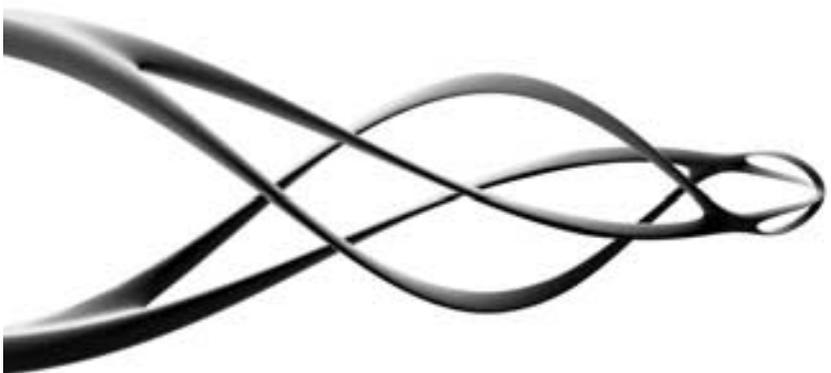


Fig. 25. Sculpture



Fractals and nature

Mandelbrot ⁷ describes the application of fractals to the derivation of form. The fractal image was produced by the successive refinement of a square grid of points on plan in which the height of each new point is the weighted average of the surrounding existing points plus a random number times the current grid spacing.



Fig. 26. Fractal mountains

The British Museum Great Court Roof



Fig. 27. British Museum Great Court Roof, Foster and Partners, Buro Happold, Waagner-Biro

The algorithm used for the geometric design of the British Museum Great Court Roof (figure 27) used a number of different types of rule. Initial studies (figure 28) used the relationship, (Green and Zerna⁸) $w = \epsilon^{\alpha\beta} \epsilon^{\lambda\mu} z_{,\alpha\lambda} \phi_{,\beta\mu}$, between the load, w , the stress function, ϕ , and the vertical coordinate, z , to derive an 'optimum' structural form. However, this approach was abandoned because other constraints could not be accommodated.

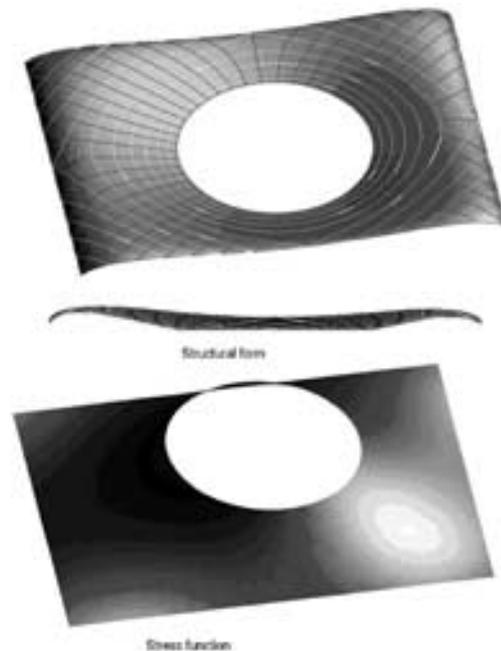


Fig. 28. British Museum Great Court Roof, initial stress function studies

The final form is described by the three functions,

$$z = \frac{h \left(1 - \frac{x}{b}\right) \left(1 + \frac{x}{b}\right) \left(1 - \frac{y}{c}\right) \left(1 + \frac{y}{d}\right)}{\left(1 - \frac{ax}{rb}\right) \left(1 + \frac{ax}{rb}\right) \left(1 - \frac{ay}{rc}\right) \left(1 + \frac{ay}{rd}\right)},$$

$$z = H \left(1 - \frac{x}{b}\right) \left(1 + \frac{x}{b}\right) \left(1 - \frac{y}{c}\right) \left(1 + \frac{y}{d}\right) \left(\frac{r}{a} - 1\right) \text{ and}$$

$$\eta \left(\frac{r}{a} - 1\right)$$

$$z = \left(\frac{\sqrt{(b-x)^2 + (c-y)^2}}{(b-x)(c-y)} + \frac{\sqrt{(b+x)^2 + (c-y)^2}}{(b+x)(c-y)} \right) + \left(\frac{\sqrt{(b-x)^2 + (d+y)^2}}{(b-x)(d+y)} + \frac{\sqrt{(b+x)^2 + (d+y)^2}}{(b+x)(d+y)} \right)$$

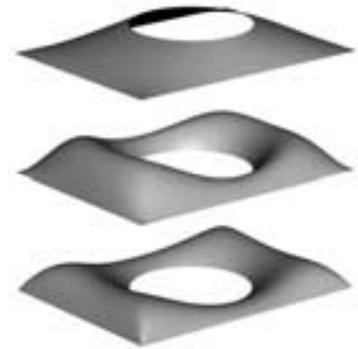


Fig. 29. British Museum Great Court Roof, functions describing surface

weighted and added together. x , y and z are the Cartesian axes, $r = \sqrt{x^2 + y^2}$, and all other quantities are constants (figure 29). The weighting functions also vary with position in plan. The first function gives the change in level between the circular Reading Room boundary and the outer rectangular boundary. The second two functions differ mainly in their behaviour at the corners, one is smooth and the other gives a concentration of curvature. This was important for the structural action – the roof is supported on sliding bearings and exerts no horizontal thrust on the existing building.

The position of the nodes of the steelwork grid upon this surface was determined by a relaxation process applied to a 'numerical grid'. The coarser structural grid is obtained by joining diagonal nodes of the numerical grid. The relaxation process involved moving each of the nodes on the numerical grid until it was the weighted average of the surrounding nodes. This process was repeated for the whole grid a large number of times, until the grid stopped moving. The weighting functions varied with position, mainly to try and limit the maximum size of glass panel. Figure 30 shows the grid before relaxation and figure 31 after relaxation.

Once this process was complete the structure was analysed in a number of ways – including the application of a stress function corresponding to the roof trying to work in compression and tension only (figure 32). However sharp folds indicated that this is not possible and therefore significant bending and torsional moments are to be expected in the structure – as confirmed by more conventional analysis methods. Figure 33 shows the roof in a collapsed state, one of many such studies which were performed as the design progressed.

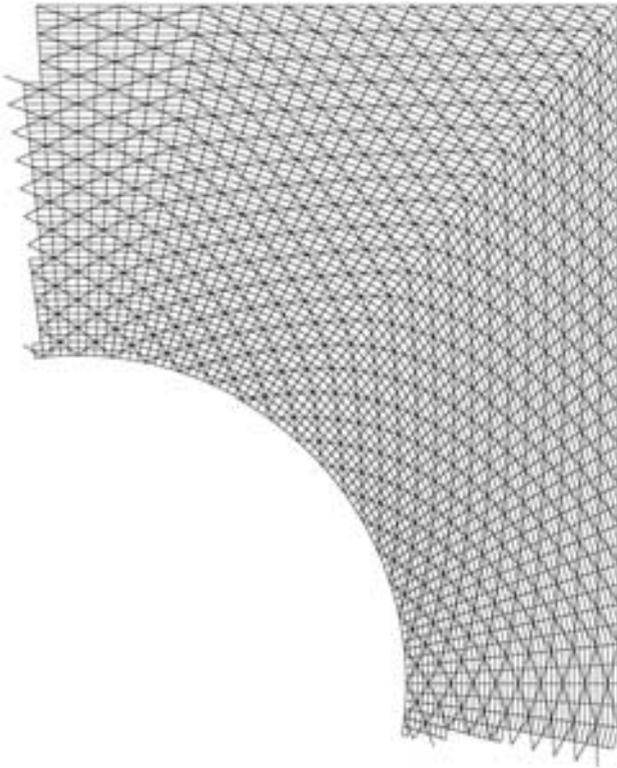


Fig. 30. British Museum Great Court Roof, grid before relaxation

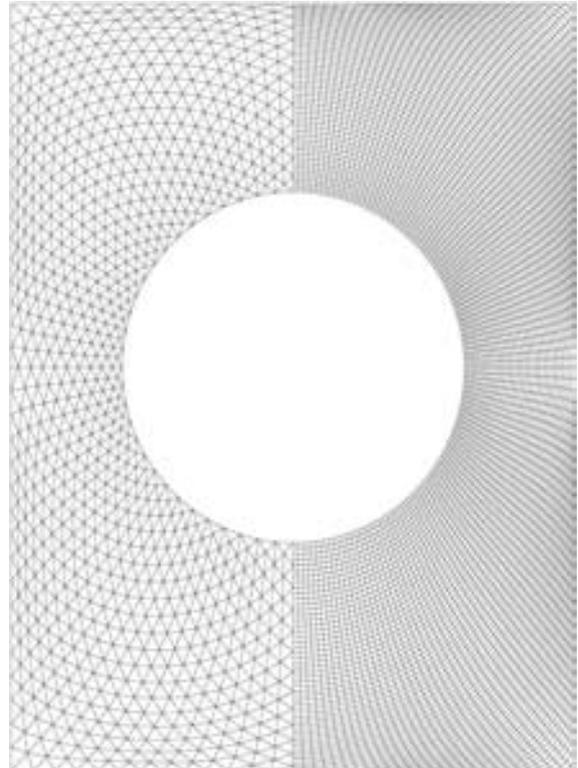


Fig. 31. British Museum Great Court Roof, grid after relaxation

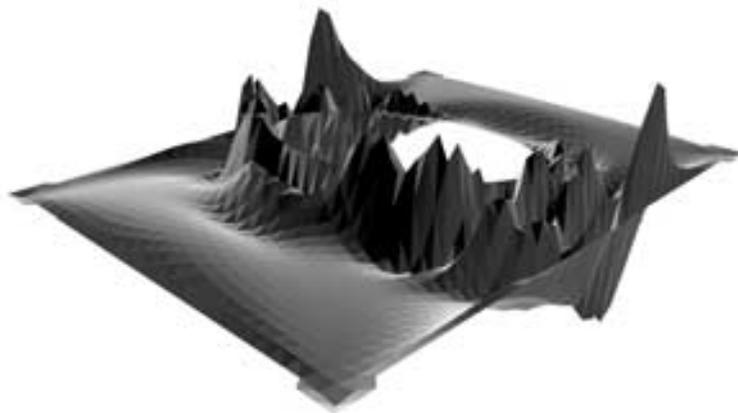


Fig. 32. British Museum Great Court Roof, final stress function studies

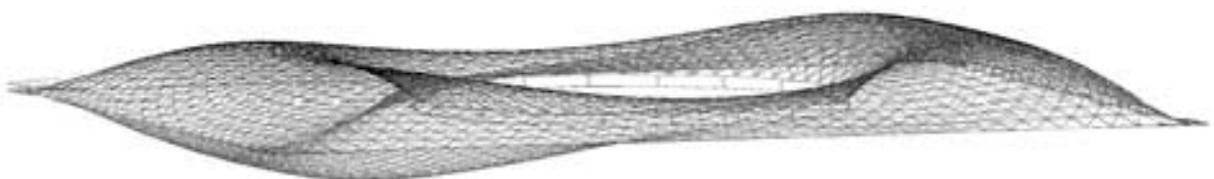


Fig. 33. British Museum Great Court Roof, collapse mode

It is difficult to know exactly what conclusions to draw. There is no doubt that objects that are designed are influenced by the design process itself. Now designs must be influenced by the computer software that is produced by people other than engineers and architects and by companies who have to respond to the market. I suppose all that I am saying is that some individual engineers and architects should be encouraged to write their own software if they are to maintain control over the design process.

Conclusion

References

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