Architectural freeform structures from single curved panels

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Abstract

The problem of covering a freeform surface by single curved panels can be treated with the concept of semi-discrete surface representations, which constitute a link between smooth and discrete surfaces. A surface composed from developable strips (called a D-strip model) is the semi-discrete equivalent of a quad mesh with planar faces, or a conjugate parametrization of a smooth surface. Using recent progress on the geometry and computation of D-strip models, we investigate their use for the segmentation into panels, for multi-layer constructions and for the supporting beam layout and manufacturing in architectural freeform structures.

Keywords: architectural geometry, discrete differential geometry, freeform surface, panelization, developable surface, developable strip model, single-curved panel, multi-layer structure, offset.

1 Introduction

Complex freeform structures are one of the most striking trends in contemporary architecture. Pioneered by F. Gehry, architects nowadays exploit digital technology originally developed for the automotive and airplane industry for tasks of architectural design and construction. This is not a simple task at all, since the architectural application differs from the original target industries in many ways, including aesthetics, statics, scale and manufacturing technologies.

Whereas metal forming can generate any reasonable shape of a car body, it is much less clear how to actually construct a complicated geometric shape in an architectural design. One has to segment the shape into simpler parts, so-called panels. Since available CAD software does not cover this topic, one may have to resort to simpler shapes, to accept higher costs or to try experimental approaches.

Very recent research shows that the use of advanced tools from mathematics and geometry processing makes a real difference in this field. An example is provided by covering freeform shapes with planar quadrilateral panels; such planar quad panels possess a number of important advantages over triangular panels: the resulting structure has a smaller number of edges, resulting in a smaller number of supporting beams following the edges, less steel and less cost; quad meshes also have a lower node complexity, which is an important advantage for manufacturing.

Panelization with planar quads can be made accessible with methods from modern discrete differential geometry [Bobenko and Suris 2005; Liu et al. 2006; Pottmann et al. 2008] and in section 3 we show a few of the many ways in which this basic theory can be applied in the actual construction of architectural freeform structures.

2 Developable strip models

A surface which is composed of developable strips may be obtained as the limit of a quad mesh with planar faces (PQ mesh) in a refinement procedure where only the rows (or the columns) get refined; see Fig. 1. Refining a PQ mesh in both directions, one obtains a so-called conjugate curve network on a smooth surface [Liu et al. 2006]. From this perspective, surfaces composed of developable strips – called D-strip models henceforth – may be viewed as a semi-discrete surface representation, which constitutes a link between smooth and discrete surfaces.

There is previous work dealing with piecewise developable surfaces: Subag and Elber [2006] approximate NURBS surfaces by piecewise developables. Several algorithms have been proposed for the construction of papercraft models [Mitani and Suzuki 2004; Massarwi et al. 2007; Shatz et al. 2006]. These contributions do not aim at smoothness of boundaries and even widths of developable pieces; consequently they are not required to exploit the semi-discrete viewpoint and the relation to conjugate curve networks and PQ meshes.

We will describe here only very briefly the computation and basic geometry of D-strip models and refer to [Pottmann et al. 2008] for more details.

Parametric representation of D-strip models. A D-strip model consists of D-strips $\mathcal{D}$, parameterized by $x_i(u,v)$, and joined together along edge curves $p_i(u)$ as shown by Fig. 1. We describe the edge curves as B-spline curves and thus the D-strips as ruled B-spline surfaces,

$$p_i(u) := \sum_j B^3(u-j) b_{ij},$$

$$x_i(u,v) := (1-v)p_i(u) + v p_{i+1}(u).$$

Here $B^3$ is the cubic B-spline basis function for integer knots.
In order to approximate a given surface $\Phi$ by a D-strip model, we compute the control points $b_{i,j}$ in an optimization algorithm by minimizing the target functional

$$
\lambda_1 f_{\text{dev}} + \lambda_2 f_{\text{prox}} + \lambda_3 f_{\text{prox}}^\partial + \lambda_4 f_{\text{fair/edge}} + \lambda_5 f_{\text{fair/ruling}}.
$$

(2)

Its individual terms measure developability of the strips, closeness to $\Phi$, closeness to the boundary curve $\partial \Phi$ if necessary, and fairness. Developability of the final surface has the nature of a constraint, which is achieved by letting $\lambda_1$ grow during iterative optimization.

The individual terms are defined as follows. Developability of the surface is expressed by a small value of

$$
f_{\text{dev}} = \sum_i \int \delta_{p_i,p_{i+1}}(u)^2 du.
$$

Here, the integrand denotes the squared distance of diagonals in the quad $(p_i, p_{i+1}, p_{i+1}, p_i)$, where dots indicate differentiation with respect to $u$. Those quads have to be planar for a developable surface. To give the distance a useful meaning, we choose $\lambda_1 = \|p_{i+1} - p_i\|/\|p_i\|$ and $\mu_i = \|p_{i+1} - p_i\|/\|p_{i+1}\|$. Proximity to a reference surface $\Phi$ is guided by

$$
f_{\text{prox}} = \sum_k \text{dist}(x_k, T_k)^2.
$$

Here, $x_k$ are sufficiently dense sample points on the strip model and $T_k$ are the tangent planes of the reference surface $\Phi$ at the points $y_k \in \Phi$ which are closest to $x_k$. Hence, we minimize squared tangent plane distances, which is known to yield better convergence than employing squared distances $\|x_k - y_k\|^2$ to closest points. For measuring distance to the boundaries of $\Phi$, we use tangents $t_k$ at boundary curves instead of tangent planes,

$$
f_{\text{prox}}^\partial = \sum_k \text{dist}(x_k, t_k)^2.
$$

For certain applications it is reasonable to approximate discrete reference points $y_j$ by the edge curves $p_i$ instead of a smooth surface $\Phi$, e.g., if one has laid out a supporting structure with fixed mounting points beforehand. In this case we employ the target functional

$$
f_{\text{prox,discrete}} = \sum_j \text{dist}(y_j, t_j)^2,
$$

where $t_j$ denotes tangents at points $p_i((u_j)$ which are closest to $y_j$. Fairness is measured with linearized bending energies of edge curves and ruling polygons:

$$
f_{\text{fair/edge}} = \sum_i \int \|\dot{p}_i(u)\|^2 du,
$$

$$
f_{\text{fair/ruling}} = \int \left( \sum_i \|p_{i+1} - 2p_i + p_{i-1}\|^2 \right) du.
$$

The iterative optimization algorithm is based on a Gauss-Newton method with Levenberg-Marquardt regularization.

**Initializing optimization.** There is a close relation between PQ meshes, D-strip models and conjugate curve networks. Therefore it is feasible to initialize the control point mesh either with a PQ mesh approximating $\Phi$ or a conjugate curve network of $\Phi$. As an example for the second possibility we consider the following common architectural problem:

Given is a family of planar and parallel sections $c_i$ of $\Phi$, which should be approximated by the edge curves $p_i$ of the D-strip model. This amounts to prescribing one family of curves of a conjugate curve network. Developability of a resulting D-strip is characterized by constant tangent planes along rulings. Thus it is reasonable to initialize the control point mesh by points on $c_i$, corresponding by parallel curve tangents. Figure 2 shows a real example utilizing floor slabs as sections.

**Principal strip models.** When approximating a surface by a D-strip model, it is natural to let edge curves follow the principal curvature lines of maximal curvature and to place rulings along the directions of the smaller principal curvature. Rather than first computing principal curvature lines and then deriving a D-strip model, we can work within the semi-discrete setting and define **principal strip models** (circular and conical models) which may be seen as limits of circular and conical meshes. These models possess remarkable geometric properties which are important for the actual architectural application (see section 3).

For brevity, we confine here to conical models. Recall that a PQ mesh is **conical** if all vertices have an associated right circular cone which is tangent to the faces adjacent to that vertex. By refinement in one direction, we get the semi-discrete version:

**Conical strip models** (Fig. 2): Each point $p_i(u)$ of an edge curve is the vertex of a rotational cone which is tangent to the two adjacent D-strips along their rulings. Hence, the tangent forms the

**Figure 2: Szervita Square, Budapest. A project designed by Zaha Hadid Architects. Example of approximating the outer shell by a D-strip model aligned with planar, parallel sections given by the three lowermost floor slabs. Sections, corresponding points used for initialization and the resulting D-strip model are shown from top to bottom. The D-strip models may be used for a further approximation using flat panels and cylinders.**
same angle with these two rulings. Optimization towards conical strip models makes use of a geometry functional which penalizes deviation from this angle equality.

The axis (properly normalized direction vector \( n_i(u) \), see [Pottmann et al. 2008] for details) of the cone with vertex \( p_i(u) \) plays the role of a surface normal. A conical model \( M \) possesses offsets \( M^d \) with edge curves \( p_i(u) + d n_i(u) \); rulings and tangent planes of \( M^d \) lie at constant distance \( d \) from the rulings / tangent planes of \( M \). The ruled surfaces (sets of cone axes) which connect corresponding edge curves of \( M \) and \( M^d \) are developable (Fig. 2, right).

It is shown in [Pottmann et al. 2008] that analogous properties hold for circular strip models. Moreover, conical and circular models are closely related and can be converted into each other by simple constructions.

**Geodesic strip models.** A geodesic curve \( c \) on a surface \( \Phi \) is a (locally) shortest path on \( \Phi \) and therefore it is also a geodesic on the developable surface \( D \) tangent to \( \Phi \) along \( c \). The geodesic curve \( c \) is mapped to a straight line in the planar unfolding of \( D \). If we glue a straight paper strip onto a physical surface model it follows along a geodesic and therefore geodesics may guide the alignment of wooden panels (Fig. 4, left) or other panels with a nearly straight development.

A geodesic curve on a smooth surface \( \Phi \) has osculating planes orthogonal to \( \Phi \). In the semi-discrete case, we therefore define that a D-strip model is a geodesic model, if the osculating planes of edge curves bisect adjacent strips. Note that such bisector planes are reasonable planes “orthogonal” to the strip model (which is itself not smooth); if the strip model converges to a smooth surface, those planes converge to exactly orthogonal planes. Each edge curve of a geodesic model has oppositely equal geodesic curvatures with respect to adjacent strips. Consequently, developing these strips yields oppositely congruent boundaries (see Fig. 4). The properties of strips imply that the development of the single strips is approximately straight. It seems feasible to cut them out of long rectangular panels. Typically a freeform surface is covered not by one, but by several geodesic D-strip models (see Fig. 5).

For Fig. 5, optimization was initialized by conjugate curve networks, where one curve family consists of geodesics. Optimization employed a term for well distributed strip widths. For the geodesic property, we used a functional which penalizes deviation of the edge curves’ osculating planes from the bisector planes of adjacent strips.

### 3 Architectural structures with skins from single curved panels

The geometric properties of D-strip models, in particular principal models, give rise to a variety of possibilities for the realization of architectural freeform structures with single curved panels. We focus here on two topics only: (i) multi-layer constructions and (ii) supporting beam layout.

Given a conical model \( M \), we may segment it into single curved patches via edge curves and selected ruling polygons. Connecting these patch boundaries with the corresponding ones on an offset model \( M^d \), we obtain a “box shell structure” composed of curved boxes each of which is bounded by two planar faces and four developable patches (see Fig. 6). The faces which connect \( M \) and \( M^d \) are suitable for the layout of supporting beams. The strips on \( M \) (and maybe also \( M^d \)) may be covered by actual panels. Exploiting manufacturing tolerances, one can try to approximate the individual developable patches by simpler ones, namely cylinders or cones.

The close relation between PQ meshes and D-strip models can be exploited to compute multi-layer structures which exhibit both types, e.g. a PQ mesh for the beam layout and a D-strip model attached to it for the actual panels. Especially if the two principal curvatures of the design surface are not too different, one may consider a structure composed of two D-strip models where the discrete direction of one strip model is aligned with the smooth direction of the other model and vice versa (Fig. 7).

Offsets also simplify the beam layout and manufacturing (Fig. 8): For any conical model \( M = \{ p_i \} \) with strips \( \mathcal{D}_i \), there are developable strips \( \mathcal{M}_i \) which connect corresponding edge curves \( p_i \) and \( p_i^d \) of \( M \) and an offset model \( M^d \). We let the stem of a curved I-beam follow \( \mathcal{M}_i \), while its horizontal bars follow D-strips orthogonal to \( \mathcal{M}_i \) (as shown in [Pottmann et al. 2008] these D-strips may...
Figure 6: Box shell structure derived from an offset pair of conical strip models.

Figure 7: Multi-layer structure formed by two D-strip models interpolating parallel PQ meshes.

be obtained via conversion to a circular model. Glass panels are aligned with further offset models. A technique for mounting the panels is sketched in Fig. 9.

Figure 8: Positioning a curved I-beam with developable stem and developable horizontal bars along the edge curve of a conical model.

Figure 9: Connecting the panels to the supporting beam.

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References


